

Logic 1

Jack Betteridge

2/8/14

Propositional Rules of \mathcal{F}_T :

• Conjunction Introduction

$$\frac{\begin{array}{c} n_1. P_1 \\ \vdots \\ n_k. P_k \\ \vdots \end{array}}{\triangleright m. P_1 \wedge \dots \wedge P_k} \quad \wedge \text{Intro: } n_1, \dots, n_k$$

• Conjunction Elimination

$$\frac{\begin{array}{c} n. P_1 \wedge \dots \wedge P_i \wedge \dots \wedge P_n \\ \vdots \end{array}}{\triangleright m. P_i} \quad \wedge \text{Elim: } n$$

• Disjunction Introduction

$$\frac{\begin{array}{c} n. P_i \\ \vdots \end{array}}{\triangleright m. P_1 \vee \dots \vee P_i \vee \dots \vee P_n} \quad \vee \text{Intro: } n$$

• Disjunction Elimination

$$\frac{\begin{array}{c} n. P_1 \vee \dots \vee P_n \\ \vdots \\ \begin{array}{c} | a_1. P_1 \\ | \vdots \\ | a_k. S \end{array} \\ \vdots \\ \begin{array}{c} | r_1. P_n \\ | \vdots \\ | r_j. S \end{array} \\ \vdots \end{array}}{\triangleright m. S} \quad \vee \text{Elim: } n, a_1-a_k, r_1-r_j$$

• Negation Introduction

$$\frac{\begin{array}{c} | a_1. P \\ | \vdots \\ | a_k. \perp \end{array}}{\triangleright m. \neg P} \quad \neg \text{Intro: } a_1-a_k$$

• Negation Elimination

$$\frac{\begin{array}{c} n. \neg \neg P \\ \vdots \end{array}}{\triangleright m. P} \quad \neg \text{Elim: } n$$

• \perp Introduction

$$\frac{\begin{array}{c} n. P \\ \vdots \\ m. \neg P \\ \vdots \end{array}}{\triangleright l. \perp} \quad \perp \text{Intro: } n, m$$

• \perp Elimination

$$\frac{\begin{array}{c} n. \perp \\ \vdots \end{array}}{\triangleright m. P} \quad \perp \text{Elim: } n$$

• Conditional Introduction

$$\frac{\begin{array}{c} | a_1. P \\ | \vdots \\ | a_k. Q \end{array}}{\triangleright m. P \rightarrow Q} \quad \rightarrow \text{Intro: } a_1-a_k$$

- Conditional Elimination

$$\begin{array}{l|l} n. P \rightarrow Q \\ \vdots \\ m. P \\ \vdots \\ \triangleright l. Q \end{array} \quad \rightarrow \mathbf{Elim: } n, m$$

- Biconditional Introduction

$$\begin{array}{l|l} \begin{array}{l|l} a_1. P \\ \vdots \\ a_k. Q \\ \vdots \\ b_1. Q \\ \vdots \\ b. P \\ \vdots \end{array} \\ \triangleright m. P \leftrightarrow Q \end{array} \quad \leftrightarrow \mathbf{Intro: } a_1\text{-}a_k, b_1\text{-}b_j$$

- Biconditional Elimination

$$\begin{array}{l|l} n. P \leftrightarrow Q \\ \vdots \\ m. P \\ \vdots \\ \triangleright l. Q \end{array} \quad \leftrightarrow \mathbf{Elim: } n, m$$

- Reiteration

$$\begin{array}{l|l} n. P \\ \vdots \\ \triangleright m. P \end{array} \quad \mathbf{Reit: } n$$

First-order Rules of \mathcal{F} :

- Identity Introduction

$$\triangleright n. x = x \quad = \mathbf{Intro}$$

- Identity Elimination

$$\begin{array}{l|l} n. P(x) \\ \vdots \\ m. x = y \\ \vdots \\ \triangleright l. P(y) \end{array} \quad = \mathbf{Elim: } n, m$$

- General Conditional Proof

$$\begin{array}{l|l} \begin{array}{l|l} a_1. \boxed{c} P(c) \\ \vdots \\ a_k. Q(c) \end{array} \\ \triangleright \forall x(P(x) \rightarrow Q(x)) \end{array}$$

- Universal Introduction

$$\begin{array}{l|l} \begin{array}{l|l} a_1. \boxed{c} \\ \vdots \\ a_k. P(c) \end{array} \\ \triangleright m. \forall x P(x) \end{array} \quad \forall \mathbf{Intro: } a_1\text{-}a_k$$

- Universal Elimination

$$\begin{array}{l|l} n. \forall x S(x) \\ \vdots \\ \triangleright m. S(c) \end{array} \quad \forall \mathbf{Elim: } n$$

- Existential Introduction

$$\begin{array}{l|l} n. S(c) \\ \vdots \\ \triangleright m. \exists x S(x) \end{array} \quad \exists \mathbf{Intro: } n$$

- Existential Elimination

$$\begin{array}{l|l} n. \exists x S(x) \\ \vdots \\ \begin{array}{l|l} a_i. \boxed{c} S(c) \\ \vdots \\ a_k. Q \end{array} \\ \vdots \\ m. Q \end{array} \quad \exists \mathbf{Elim: } n, a_1\text{-}a_k$$