Classical Mechanics
&
Special Relativity

Jack Betteridge

5/8/14

Galilean Transformations: DIAGRAM

$$\begin{aligned} t' &= t \\ x' &= x - ut \\ y' &= y \\ z' &= z \end{aligned} \quad \rightleftarrows \quad \begin{aligned} t &= t' \\ x &= x' + ut \\ y &= y' \\ z &= z' \end{aligned}$$

$$v = v' + u \quad \rightleftarrows \quad v' = v - u$$

Lorentz Transformations: DIAGRAM

$$\gamma = \gamma(u) = \frac{1}{\sqrt{1 - u^2/c^2}}$$

$$\begin{aligned} t' &= \gamma \cdot (t - ux/c^2) \\ x' &= \gamma \cdot (x - ut) \\ y' &= y \\ z' &= z \end{aligned} \quad \rightleftarrows \quad \begin{aligned} t &= \gamma \cdot (t' + ux/c^2) \\ x &= \gamma \cdot (x' + ut') \\ y &= y' \\ z &= z' \end{aligned}$$

Length Contraction: $L = L_0/\gamma$ (in x-direction)
Time Dilation: $\Delta t = \gamma \cdot \Delta t_0$
Doppler Effect: DIAGRAM
$\Delta t_{Obs} = \Delta t \pm \frac{u}{c}\Delta t = \gamma \cdot (1 \pm u/c) \cdot \Delta t_0$
$f = 1/t_{Obs}$, so $\frac{f}{f_0} = \frac{1}{\gamma \cdot (1 \pm u/c)}$ also, $\lambda = 1/f$
$+$ for a source moving away, $-$ for towards.

Also:

$$v = \frac{dx}{dt} = \frac{dx}{dt'}\frac{dt'}{dt} = \frac{dx}{dt'}\frac{1}{\frac{dt}{dt'}}$$

For 1D:

$$v = \frac{v' + u}{1 + uv'/c^2} \quad \rightleftarrows \quad v' = \frac{v - u}{1 - uv/c^2}$$

For 3D:

$$\left.\begin{aligned} v'_x &= \frac{v_x + u}{1 + uv'/c^2} \\ v'_y &= \frac{v_y}{\gamma \cdot (1 - uv/c^2)} \\ v'_z &= \frac{v_z}{\gamma \cdot (1 - uv/c^2)} \end{aligned}\right\} \rightleftarrows \left\{\begin{aligned} v_x &= \frac{v'_x + u}{1 + uv'_x/c^2} \\ v_y &= \frac{v'_y}{\gamma \cdot (1 + uv'_y/c^2)} \\ v_z &= \frac{v'_z}{\gamma \cdot (1 + uv'_z/c^2)} \end{aligned}\right.$$

Mass: $m = \gamma \cdot m_0$
Momentum: $\rho = \gamma \cdot m_0 \mathbf{v}$

[Force: $\mathbf{F} = \frac{d\rho}{dt} = \gamma \cdot m_0\mathbf{a} + \dot{\gamma} \cdot m_0\mathbf{v}$ N.B: $\mathbf{F} \neq m\mathbf{a}$, not even necessarily parallel.]
Energy: $\Delta E = \Delta m_0 c^2$
$E = E_0 + E_K = \gamma \cdot m_0 c^2$
$E_K = (\gamma - 1)m_0 c^2$
Energy & Momentum: $E^2 - \rho^2 c^2 = m_0^2 c^4$ (A Lorentz Invarient)
Massless Particles: $E = \rho c$
$s^2 = c^2t^2 - x^2 - y^2 - z^2$ & $\Delta s^2 = c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$ are also Lorentz invarients.

Newton's Second Law: $\mathbf{F} = \frac{d\rho}{dt} = \frac{d(m\mathbf{v})}{dt}$
Newton's Law of Gravitation: $\mathbf{F} = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$
Centre of Mass: $\bar{\mathbf{r}} = \frac{\sum m\mathbf{r}}{\sum m}$
Static Friction:$F_s \leq \mu_s N$
Static Friction on an Inclined Plane: $\tan\theta \leq \mu_s$
Kinetic Friction: $F_K = \mu_K N$
Rolling Friction: $F_R = \mu_R N$

Fluid Forces: (k arbitrary)
Floatation Force: $\mathbf{F} = k \times \rho L^3 g$
Air Resistance: $k \times \rho L^2 v^2$
Viscous Drag: $k \times \eta L v$ ($[\eta] = ML^{-1}T^{-1}$–viscosity)
Pressure Force: $k \times PL^2$
Equation of Hydrostatic Equilibrium: $\frac{dP}{dz} = -\rho g \implies$
Pressure at Depth h: $P = \rho g h$
Pressure of an Ideal Gas: $P = \frac{k_B T}{\mu}\rho$

Delta V: (Rockets) $v_e \log(\frac{m_0}{m})$
Delta V (at Launch): $v_e \log(\frac{m_0}{m}) - gt$

Angular Speed: $\omega = \frac{d\theta}{dt}$
Speed: $v = \omega r$
Acceleration: $a = \omega^2 r = \frac{v^2}{r} \quad \overset{F=ma}{\Longrightarrow}$
Force: $F = \frac{mv^2}{r} = m\omega^2 r$

Velocity: $\mathbf{v} = \omega \times \mathbf{r}$
For Orbit: $\omega^2 = \frac{4\pi^2}{T_p^2} = \frac{GM}{R^3}$ (T_P–Time Period)
Torque: $\tau = \mathbf{r} \times \mathbf{F}$, $\tau = \frac{d\mathbf{L}}{dt}$
Angular Momentum: $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, $L = I\omega$
Moment of Inertia: $I = \sum mr^2 = \int r^2.dm = \int r^2 \rho.dV$
Sometimes (When???): $\mathbf{L} = I\omega$, $\tau = \frac{d\mathbf{L}}{dt} = I\frac{d\omega}{dt}$

Common Moments of Inertia:
Ring: $I = MR^2$
Disc: $I = \frac{1}{2}MR^2$
Sphere: $I = \frac{2}{5}MR^2$
Rod (through centre): $\frac{1}{12}ML^2$
Rod (through end): $\frac{1}{3}ML^2$
When no external torques act angular momentum is conserved: $\tau = 0$

Work Done: $\Delta W = \int_A^B \mathbf{F}.d\mathbf{r}$
Kinetic Energy (Linear): $E_K = \frac{1}{2}mv^2$
Kinetic Energy (Rotational): $E_K = \frac{1}{2}I\omega$
Kinetic Energy (Total): $E_K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Work Done by a conservative force: $\Delta W = \int_A^B \mathbf{F}.d\mathbf{r} = U(A) - U(B) = -\Delta U$, so $E = E_K + U = \text{constant}$
Work you do: $\Delta W_{YOU} = \Delta U$
Gravitational Potential Energy: $U(r) = -\frac{GMm}{r}$
Gravitational Potential: $u(r) = \frac{U(r)}{m} = -\frac{GM}{r}$
Escape Velocity: $v > \sqrt{\frac{2GM}{R}}$
Power: $P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v} = \tau \cdot \omega$

Effective Potential: $E = E_K + U = \frac{1}{2}m(v_r^2 + v_\theta^2) + U$
$\epsilon = \frac{E}{m} = \frac{1}{2}\dot{r}^2 + \frac{1}{2}(r\dot{\theta}) + u(r)$
$L = r(mv_\theta) = mr^2\dot{\theta}$
$r^2\dot{\theta} = \frac{L}{m} = h$
$\frac{1}{2}\dot{r}^2 + \frac{h^2}{2r^2} + u(r) = \varepsilon$
or $\frac{1}{2}\dot{r}^2 + u'(r) = \varepsilon$
Effective Potential: $u'(r) = u(r) + \frac{h^2}{2r^2}$
Polar Equation Ellipse: $r = \frac{\ell}{1+e\cos\theta}$
Semi-major axis: $a = \frac{\ell}{1-e^2}$
Orbit under gravity: $\frac{d^2u}{d\theta^2} + u = \frac{GM}{h^2}$
Kepler's First Law: $\ell = \frac{h^2}{GM}$
Kepler's Third Law: $\frac{4\pi^2}{T_p P^2} = \frac{GM}{a^3}$
Orbital Energy per unit mass: $\varepsilon = -\frac{GM}{2a}$
Binary System: $\frac{4\pi^2}{T_P^2} = \frac{G(M_1+M_2)}{a^3}$